Ship motion and stability in longitudinal seaways – A simulation analysis

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ABSTRACT

Dynamics and stability characteristics of ships are specific to the ship configuration, the sea condition and the sailing direction. A computer-based method for determining the hydrodynamic force co-efficients from the ship hull geometry is developed. Such co-efficients appear in non-linear coupled differential equations which describe the ship's motion. The analysis assumes a constant velocity for the ship moving in a longitudinal seaway.

Fishing vessels, in particular, are vulnerable to seaway conditions. Due to significant weight variation between empty and fully loaded conditions, and also due to possible uneven load distribution, the characteristic range of such ships is wide, making the design and analysis difficult. Experimental methods are valuable tools and provide a realistic insight into the problem. Such tools, however, are limited in the sense that it is uneconomical to construct physical models of all the different designs being evaluated.

After the initial hydrodynamic force co-efficients are determined from hull geometry, a digital simulation analysis provides the response characteristics to variations in design parameters and operating conditions.

Digital computer simulation is performed and the mechanism for instability and capsizing is investigated. Unstable behaviour is found to be closely related to waves of lengths of similar magnitude to the ship length. The effect of wave amplitude is found to be significant in the capsizing process.

Using a commercially available simulation language, the work is being continued towards development of a more generalized interactive ship stability analysis package.

INTRODUCTION

The problem of ship motions and ship stability is of fundamental concern to naval architects during the design process. Conventional techniques and regulations have been reasonably successful in assuring a satisfactory level of stability for large vessel types, however the number of ship losses, especially those of medium and small size, require new methods of analysis.

Although experiments provide a realistic insight of the significant factors involved in a given physical event, they are, in general, expensive and time consuming. Therefore, an alternative method should be developed in order to predict, in a faster and relatively inexpensive form, the characteristic factors of that particular phenomenon. This becomes of capital importance in the optimization process followed in the design spiral of a ship.

The study of the seakeeping characteristics of ships has been a major area of research among the naval architecture community in the last 40 years. A complete investigation of this phenomenon would require an analysis of the compounded effect of the steering mechanism and the ship's nonlinear motions in all six coupled degrees of freedom - surge, sway, heave, roll, pitch and yaw - in realistic complex seas of multiple directionality. Such a complex and complete analysis has not yet been made. As a first step, the simplified porblems of ship motions have been tackled by several researchers.

Kriloff [1896,1898] developed the "strip theory" for computing pitch and heave motions of a ship in regular waves. Weinblum and St. Denis [1950] made the first attempt to define ship motions when advancing on an arbitrary course. St. Denis [1951] and St. Denis and Pierson [1953] further improved this work to account for an irregular seaway. Korvin-Kroukovsky [1955] and Korvin-Kroukousky and Jacobs [1957] included pitch-heave coupling and an approximate calculation of ship-wave interaction.

All these works assumed motion and wave amplitudes to be small and neglected the possibility of rolling in head or following seas. Grim [1952] showed that for certain values of the encounter frequency ω_e, rolling instability could develop. Kerwin [1955] solved the roll equation of motion treating it as a Mathiew differential equation. Solution by expansion series showed instability occurred when the ratio of the roll natural frequency to the exciting frequency ω_e took on half integer values M/2 (M = 1, 2, 3, ...). Paulling et. al. [1959, 1961] confirmed this finding experimentally and numerically. Salvensen, Tuck and Faltinsen [1970] implemented a linear procedure for ship motion in the frequency domain. Oakley, Paulling and Wood [1974] carried out a numerical investigation of ship motions in extreme seas using constant hydrodynamic coefficients.

Using the strip theory and the Froude-Kriloff hypothesis a time domain numerical simulation of ship motion, taking into account the time dependency of the nonlinear roll damping coefficients, has been developed by Allievi [1987]. The work presented in this paper has been adopted from this reference. The computer program uses a fixed step integration method for calculating the state variables. Currently use of ACSL, Advanced Continuous Simulation Language, Mitchel and Gauthier [1981,1986], is being investigated. The significant part of calculating the hydrodynamic coefficients and forces will still be performed in FORTRAN-77. Facilities offered by ACSL such as different integration methods, accuracy controls, automatic ploting and run time command will be used to determine state variables under different strategies. Due to the three dimensional curvature of the ship, accuracy implemented through the strip theory makes the calculation of state variables through predictor-corrector routines of ACSL a time-insentive process. Preliminary runs on an Apollo DN570 for simulating a 100 second real-time-ship-motion took in the order of 24 hours of CPU time.

INSTABILITY OF ROLLING MOTION

Figure 1 shows the righting arm curve of a ship in still water. This figure also shows the effect on the GZ-curves of a wave of equal length to the ship length between perpendiculars with its crest and trough positioned amidships. The drastic change in the GZ-curves can be realized. Consequently, when a ship is sailing in a longitudinal seaway there is a periodic variation of its static stability curves corresponding to the relative position of the wave along the ship's length caused by the vertical displacement of the transverse metacenter. As a result, the initial transverse stability will vary with a period equal to the period of wave encounter.

This process is known as low cycle resonance and can be qualitatively visualized by considering the two extreme positions of the wave, that is with the crest amidships and with the crest at the ends. For conventional ship forms with flared sections at the ends and wall-sided sections amidships it can be seen that the

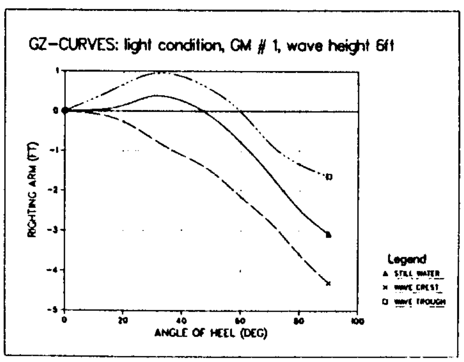

Figure 1: Righting arm curves.

inertia of the waterplane will change in each case. It is smaller than the still water value with the wave crest amidships and greater than the still water value with the wave crest at the ends. If the ship is now given an initial roll angle θ_0 caused by some disturbance, it will oscillate at its natural period until the energy contained by the disturbing moment is dissipated by damping. However, if the period of the stability fluctuation and the natural roll period are in the right ratio, it is possible that the initial roll angle be sustained and even increased to considerable proportions. This process may result in constantly increasing roll angles that may eventually lead to capsizing.

FORMULATION OF THE PROBLEM

The vessel is assumed to have a plane of symmetry as regards the shape of its lines plan. Further, it is assumed to behave as a rigid body having, initially, six degrees of freedom. Newton's second law may then be written in its linear momentum form in the following manner:

$$\frac{d}{dt}(m\,\bar{v}) = \bar{F} \tag{1}$$

Conservation of angular momentum gives:

$$\frac{d}{dt}(I\,\bar{\Omega}) = \bar{M} \tag{2}$$

The right-hand side of Eqs. (1) and (2) are the sums of the forces and moments resulting from the interaction of the ship and waves in addition to the gravitational force. Effects such as wind and currents are not considered. The motion variables involved are the instantaneous position, velocity and acceleration of the ship.

SYSTEMS OF COORDINATE AXES

For the determination of the spatial position of the moving vessel, three coordinate systems are introduced as shown in Figure 2 and Figure 3. These are:

1. O_1-$\xi_1\eta_1\zeta_1$ Newtonian system of coordinates fixed in space and so oriented that $\xi_1\eta_1$ plane lies on the undisturbed water surface.
2. G-$\xi\eta\zeta$ attached to the center of gravity of the vessel and parallel to the system O_1-$\xi_1\eta_1\zeta_1$ during the entire duration of the motion. It defines the mean translational position of the vessel with respect to O_1-$\xi_1\eta_1\zeta_1$.
3. G-XYZ system of Cartesian coordinates attached to the center of gravity of the vessel. The motion of G-XYZ with respect to G-$\xi\eta\zeta$ gives the angular displacements of the ship.

The motion of the vessel is then composed of the translational motion of its center of gravity G and the rotational motion of the axes G-XYZ about the axes G-$\xi\eta\zeta$. Therefore, the position of the ship is determined by three coordinates $G\xi_g$, $G\eta_g$, $G\zeta_g$ as shown in Figure 3 and three Eulerian angles θ, ψ and ϕ as shown in Figure 2. Then, two vectors, translation Ξ and rotation T, can be defined as follows:

$$\{\Xi\} = \{\xi_g \quad \eta_g \quad \zeta_g\} \tag{3}$$

$$\{T\} = \{\theta \quad \psi \quad \phi\} \tag{4}$$

EULER ANGLES

Euler angles may be chosen in diverse ways. As first published by Scientia Navalis of St. Petersburg in 1749 and as adopted in theoretical mechanics, Euler angles proved to be inconvenient in the study of ship motions. Kriloff [1951] proposed a new "nautical" system of Euler angles free from their initial inherent problems. This nautical system of Euler angles can in turn be chosen in different ways which, although having no effect on the final results, permit a greater or lesser degree of clarity in the visualization of the motions.

The system adopted in this work was first introduced by Blagoveshchensky [1954], and as mentioned above, is shown in Figure 2. The line GN, called nodal line, is defined by the intersection of the coordinate planes $\xi G\eta$ and YGZ. The positive direction of GN is obtained by the counterclockwise rotation of the axis ζ toward the X-axis through the smallest angle looking from this positive direction. The plane defined by XGZ is always perpendicular to the nodal line. Euler angles are defined as follows:

- Roll angle θ lies in the plane YGZ. The angle of roll is the rotation about the X-axis and positive to starboard.

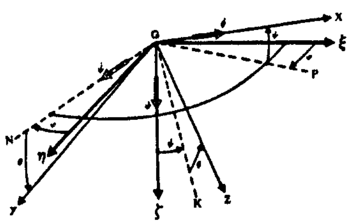

Figure 2: Euler angles.

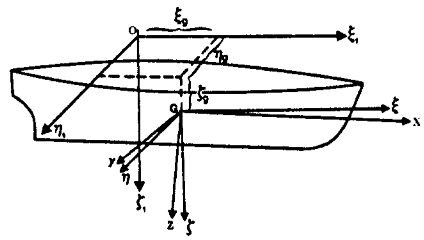

Figure 3: Coordinate systems.

- Pitch angle ψ lies in the plane ζGX. The value of ψ is obtained by subtracting 90 degrees from the angle formed by $G\zeta$ and GX.

- Yaw angle ϕ lies in the plane $\xi G\eta$. Positive values are measured counterclockwise from $G\eta$ to GN looking from the positive direction of $G\zeta$.

COORDINATE SYSTEM TRANSFORMATION

Transformation from the stationary coordinate system to the ship coordinate system is attained by means of the following expressions, Blagoveshchensky [1954]:

$$\xi_1 = \xi_g + a_1 x + b_1 y + c_1 z \tag{5a}$$

$$\eta_1 = \eta_g + a_2 x + b_2 y + c_2 z \tag{5b}$$

$$\zeta_1 = \zeta_g + a_3 x + b_3 y + c_3 z \tag{5c}$$

and visceversa:

$$x = a_1(\xi_1 - \xi_g) + a_2(\eta_1 - \eta_g) + a_3(\zeta_1 - \zeta_g) \tag{6a}$$

$$y = b_1(\xi_1 - \xi_g) + b_2(\eta_1 - \eta_g) + b_3(\zeta_1 - \zeta_g) \tag{6b}$$

$$z = c_1(\xi_1 - \xi_g) + c_2(\eta_1 - \eta_g) + c_3(\zeta_1 - \zeta_g) \tag{6c}$$

The instantaneous position of the vessel is given by the three coordinates of the center of gravity ξ_g, η_g and ζ_g and the three Eulerian angles θ, ψ and ϕ. The relationship between the system of axis attached to the ship is defined by the direction cosines a_i, b_i and c_i with $i = 1,2,3$. The equations defining the direction cosines are given in the Appendix.

EQUATIONS OF MOTION

The equations of motion of the ship, assumed to behave as a rigid body, can be divided in two groups. One group defines the translation of the center of gravity of the ship. A second group describes the rotations of the vessel about the fixed axes passing through its center of gravity.

On the basis of Newton's second law the translational equations of motion are as follows:

$$\frac{\nabla}{g}\frac{d^2}{dt^2}\xi_g = \Sigma F_{\xi_1} \tag{7a}$$

$$\frac{\nabla}{g}\frac{d^2}{dt^2}\eta_g = \Sigma F_{\eta_1} \tag{7b}$$

$$\frac{\nabla}{g}\frac{d^2}{dt^2}\zeta_g = \Sigma F_{\zeta_1} \tag{7c}$$

The right-hand side members of Eqs. (11) are the sums of all the forces acting upon the vessel projected on to the stationary axes $O_1\xi_1$, $O_1\eta_1$ and $O_1\zeta_1$.

The equations of motion for the rotational degrees of freedom were introduced in vectorial form in Eq. 2. This equation can be decomposed into three equations representing the moments about the moving axes. That is:

$$\Sigma M_x = I_{xx}\dot{p} + (I_{zz} - I_{yy})\,qr - I_{zx}pq - I_{xz}\dot{r} \tag{8a}$$

$$\Sigma M_y = I_{yy}\dot{q} + (I_{xx} - I_{zz})\,pr - I_{zx}\,(p^2 - r^2) \tag{8b}$$

$$\Sigma M_z = I_{zz}\dot{r} - I_{zx}\dot{p} + (I_{yy} - I_{xx})pq + I_{xz}rq \tag{8c}$$

where p, q and r are the orthogonal components of the vessel's angular velocity vector $\vec{\Omega}$ in the G-XYZ system. Their expressions are defined in the Appendix.

COMPUTATION OF FORCES AND MOMENTS

It was mentioned that forces and moments acting on the ship may be assumed to be predominantly hydrostatic provided that a low value of the encounter frequency is maintained. Hydrostatic forces are modelled by accurately integrating the wave pressure field around the wetted surface of the hull up to the wave surface.

Hydrodynamic coefficients for pitch and heave are obtained by using a close-fit method. Roll motion, as one of the most important responses of a ship, demands a correct prediction of roll damping in order to ensure safety as well as to attain a fair understanding of the ship motions in waves. The vessel under study was observed to possess a very high roll damping with marked nonlinear behaviour. For this reason a more detailed component analysis of the roll damping was performed following Himeno [1981].

Forces and moments due to wave action in the G-XYZ system of coordinates are:

$$F_{x,w} = -\iiint \frac{\partial P}{\partial x}\,dv \tag{9a}$$

$$F_{y,w} = -\iiint \frac{\partial P}{\partial y}\,dv \tag{9b}$$

$$F_{z,w} = -\iiint \frac{\partial P}{\partial z}\,dv \tag{9c}$$

$$M_{x,w} = -\iiint \left(z\frac{\partial P}{\partial y} - y\frac{\partial P}{\partial z}\right)dv \tag{10a}$$

$$M_{y,w} = -\iiint \left(z\frac{\partial P}{\partial z} - z\frac{\partial P}{\partial x}\right)dv \tag{10b}$$

$$M_{z,w} = -\iiint \left(y\frac{\partial P}{\partial x} - x\frac{\partial P}{\partial y}\right)dv \tag{10c}$$

Gauss Theorem is used to transform the volume integrals in Eqs. (9) and (10) into surface integrals more amenable to numerical analysis.

At this stage the computer program uses the undisturbed pressure fields derived from two-dimensional linear wave theory and nonlinear Stokes second order wave theory, Stokes [1847]. It is apparent that the flow field is modified by the motions of the ship.

These motions, which are dependent on the ship geometry and mass distribution, are also affected by the modifications of the flow field itself. A three dimensional flow pattern develops with flow separation and turbulence in particular toward the ends of the ship. However, the assumption of an undisturbed two-dimensional flow field under the condition of a low encounter period yields reasonably good agreement between experimental and numerical results.

This work is restricted to ship response in a pure longitudinal seaway. For this reason, sway and yaw motions are not considered. In case of having any obliquity between the ship's centerplane and the wave crest, consideration must be given to these two degrees of freedom as well as to a wave exciting term in the roll equation of motion. Under the assumption of a low encounter frequency value, surge may be neglected. The ship speed is then considered to be constant. The equations to be solved are then (7c), (8a) and (8b).

The total heave force is obtained in the following manner:

$$C_h = C_{h,w} - B_h \dot{\zeta}_g - R_h \zeta_g \qquad (11)$$

where

$$R_h = \rho g \int 2\, y(x)\, dx$$

The value of C_h is calculated at each time step and introduced into equation (14a). Moments due to wave action are affected by damping and restoring moments in a similar fashion as in Eq. (11). That is:

$$\Sigma M_x = M_{x,w} - B_r \dot{\theta} \qquad (12)$$

$$\Sigma M_y = M_{y,w} - B_p \dot{\psi} - R_p \psi \qquad (13)$$

The restoring moment coefficient in pitch in (13) was approximated as $\nabla\, GM_L$, Bhattacharyya [1978]. Equations (12) and (13) are calculated at each time interval and introduced in equation (15a) in place of $\bar{M}$ to carry out the numerical integration. The equations are referred to the G-XYZ system of coordinates.

NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION

The numerical integration in the time domain of the equations of motion was made using a fourth-order Runge Kutta process as developed by Gill [1951]. The equations of motion (1) and (2) are re-written as first order simultaneous ordinary differential equations in the following manner:

$$\frac{d\bar{v}}{dt} = \frac{\bar{F}}{m} \qquad (14a)$$

$$\frac{d\bar{\Xi}}{dt} = \bar{v} \qquad (14b)$$

$$\frac{d\bar{\Omega}}{dt} = I^{-1}\,[\bar{M} - \bar{\Omega} \times I\bar{\Omega}] \qquad (15a)$$

$$\frac{d\bar{T}}{dt} = [T]^{-1}\,\{\bar{\Omega}\} \qquad (15b)$$

Equations (14) and (15) give a system of six coupled simultaneous highly nonlinear first order differential equations which are solved by numerical integration in the time domain.

SIMULATION RUNS

A series of runs was performed in order to study the economical aspect of the computer program. The number of hull elements was gradually increased and the CPU time required for a fixed maximum simulation time to be achieved was measured. Figure 4 shows plots of the results. An almost linear increasing tendency in computer dollars can be realized as the hull is defined in a more accurate manner. It was found that a waterline spacing of 0.5 ft., i.e. 676 hull elements, gave accurate results at minimum cost (0.591 CPU sec/number of Δt).

In order to assess the validity of the numerical simulation a series of still water runs at zero forward speed was made and compared with available experimental data. Figure 5 shows curves of the roll decay motions of the vessel for the extreme GM configurations in heavy load conditions. Both periods and amplitudes of the motions show a very satisfactory agreement with experimental values. From these results it was concluded that the roll damping scheme realistically predicted the energy loss during the roll motion. Furthermore, exact knowledge of the initial conditions resulted in a very acceptable prediction of the motions.

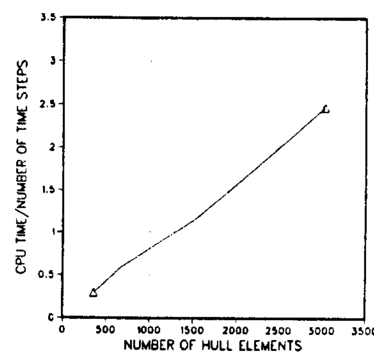

Figure 4: CPU time(sec) vs number of hull elements.

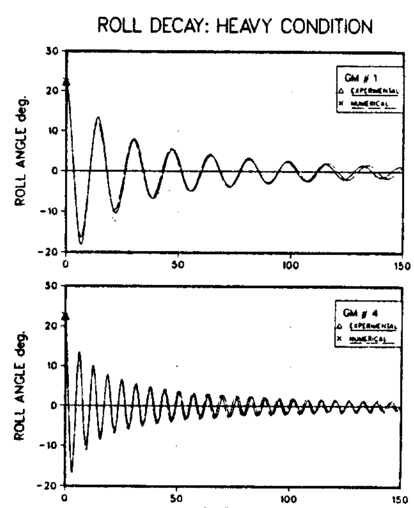

Figure 5: Roll decay curves.

For the motion of the ship in waves two examples are given in Figure 6 and Figure 7. The ship speed adopted is 9.2 knots. These are capsizing cases in which the wave height has been increased to show the effect of wave amplitude in the capsizing time. The wave length has been kept constant and equal to 92.3 ft. The initial roll angle is 0.1 degree in both cases. Wave records correspond to the wave amplitude at the vessel's centre of gravity.

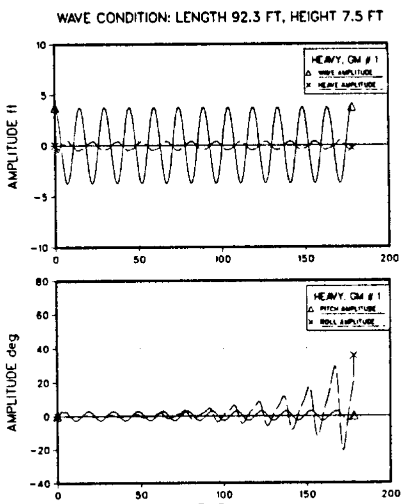

Figure 6: Capsizing case.

The motions predicted by the program follow an expected trend, in particular for the lower height case shown in Figure 6. Here, pitch and heave records clearly display a transient-state up to about 50 - 70 sec. until they become almost sinusoidal curves. In this steady state region the records corresponding to these motions show almost identical periods of oscillation and a consistent phase angle of 90 degrees. The roll record displays a typical low cycle resonance build-up. This is because the wave encounter period bears a unity critical relationship with the roll period in waves. This record also displays a "pause-type" of irregularity just before the wave crest passes by the midship section. Following the pause and as the wave crest moves into the midship section the roll angle increases. This is aggravated by the fact that the pitch angle passes from about a zero angle - when the pause occurs - to a positive value that causes bow emergence. This results in loss of waterplane area and in reduced stability. As the roll angle attains significant values pitch and heave steady-state behaviour is disturbed. This can easily be seen when capsizing occurs at about 185 seconds.

Figure 7 shows a capsizing case with very dynamic characteristics. Pitch and heave records never attain a steady-state behaviour since capsizing occurs before this stage can be reached. The roll record again displays a pause just prior to the crest reaching the midship section. A severe roll increase occurs at about 43 seconds due to the combined effects of wave crest and positive pitch. From this point onwards pitch and heave records are greatly disturbed until capsizing occurs at about 59 seconds after only four crest encounters. Experimental tests for this configuration and wave condition yielded a capsizing time of 54 seconds.

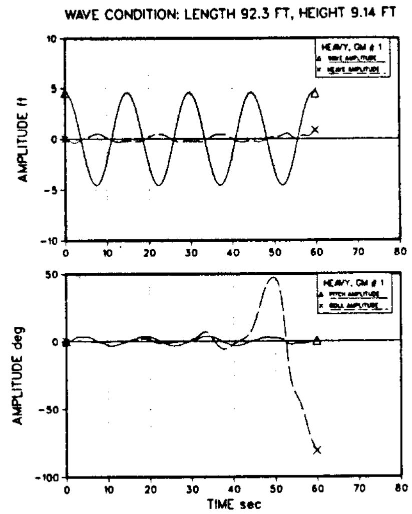

Figure 7: Capsizing case.

CONCLUSIONS

- A numerical model to predict a ship's coupled motions in a longitudinal seaway has been performed. The model is quite efficient and gives results which compare favourably over a broad range of wave frequencies with available experimental results within the limits of the implied assumptions. The model also showed to be very useful in predicting capsizing conditions.
- A time dependent analysis of the roll damping has been implemented. The nonlinearity of the roll damping coupled with its dependence on forward speed has been accounted for. The method showed effectiveness in predicting the roll motion characteristics of the stationary fishing vessel.
- Large roll motions develope within a small number of cycles and lead to capsizing. Significantly shorter roll natural periods may make the small vessel's capsizing time insufficient for seaman action to be taken.

NOMENCLATURE

a_i, b_i, c_i; i = 1,2,3 : direction cosines
A_h : added mass in heave
B : beam
B_h : hydrodynamic damping in heave
B_p : hydrodynamic damping in pitch
B_r : total hydrodynamic roll damping
C_b : block coefficient
C_h : total heave force
$C_{h,w}$: heave force due to wave action
C_m : midship-section coefficient
D : depth to main deck (molded)
$\bar{F}$: force vector
$F_{i,w}$; i = x,y,z : force due to wave action in X, Y and Z directions
g : acceleration of gravity
GM : transverse metacentric height
GM_L : longitudinal metacentric height
GZ : righting arm
I_{ii}; i = x,y,z : virtual moment of inertia about X, Y and Z axes
k : wave number
L : ship length
LBP : length between perpendiculars
m : virtual ship mass, $m = \nabla/g + A_h$
$\bar{M}$: moment vector
$M_{i,w}$; i = x,y,z : moment due to wave action about X, Y and Z axis

ΣM_x : total moment about X-axis
ΣM_y : total moment about Y-axis
ΣM_z : total moment about Z-axis
$p, \dot{p}$: angular velocity and acceleration about X-axis
P : pressure
$q, \dot{q}$: angular velocity and acceleration about Y-axis
$r, \dot{r}$: angular velocity and acceleration about Z-axis
R_h : restoring force coefficient in heave
R_p : restoring moment coefficient in pitch
T : ship draft
$[T]$: coordinate transformation matrix
T_e : wave encounter period
$\bar{v}$: translational velocity vector
x : x-coordinate of ship particle in G-XYZ system
y : y-coordinate of ship particle in G-XYZ system
$y(x)$: half breadth at cross section x
z : z-coordinate of ship particle in G-XYZ system
$\phi, \dot{\phi}$: sway angle and angular velocity
$\psi, \dot{\psi}$: pitch angle and angular velocity
$\theta, \dot{\theta}$: roll angle and angular velocity
λ : wave length
ω_r : natural roll frequency $= \sqrt{(\nabla\, GM/I_{xx})}$
ω_e : wave encounter frequency $= 2\pi/T_e$
ξ_g : coordinate of center of gravity in $O_1 - \xi_1\eta_1\zeta_1$ system
η_g : coordinate of center of gravity in $O_1 - \xi_1\eta_1\zeta_1$ system
ζ_g : coordinate of center of gravity in $O_1 - \xi_1\eta_1\zeta_1$ system
ξ_1 : coordinate of ship particle in $O_1 - \xi_1\eta_1\zeta_1$ system
η_1 : coordinate of ship particle in $O_1 - \xi_1\eta_1\zeta_1$ system
ζ_1 : coordinate of ship particle in $O_1 - \xi_1\eta_1\zeta_1$ system
∇ : displacement
Ω : angular velocity vector
Ξ : vector translation
T : vector rotation

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APPENDIX

Ship Characteristics	Light	Heavy
Length Overall (LOA)	-----	77.00 ft
Length btn. Perpendiculars (LBP)	-----	70.00 ft
Beam (B)	-----	23.00 ft
Depth (D)	-----	15.00 ft
Draft (T)	9,500 ft	10.50 ft
Displacement (∇)	220.3 ton	254.2 ton
Block Coefficient (Cb)	0.500	0.531
Midship-Section Coefficient (Cm)	0.756	0.775
Waterplane Coefficient (Cwp)	0.850	0.862
KM	13,780 ft	12,820 ft
GM #1 (1.11% B)	0.2560 ft	0.2560 ft
GM #2 (2.22% B)	0.5110 ft	0.5110 ft
GM #3 (5.00% B)	1.1520 ft	1.1520 ft
GM #4 (9.27% B)	2.1330 ft	2.1330 ft

Direction Cosines

The expressions for the direction cosines are as follows:

$a_1 = \cos(XG\xi) = \cos\phi \cos\psi$
$a_2 = \cos(XG\eta) = \sin\phi \cos\psi$
$a_3 = \cos(XG\zeta) = -\sin\psi$
$b_1 = \cos(YG\xi) = \sin\theta \cos\phi \sin\psi - \cos\theta \sin\phi$
$b_2 = \cos(YG\eta) = \cos\theta \cos\phi + \sin\theta \sin\phi \sin\psi$
$b_3 = \cos(YG\zeta) = \sin\theta \cos\psi$
$c_1 = \cos(ZG\xi) = \cos\theta \cos\phi \sin\psi + \sin\theta \sin\phi$
$c_2 = \cos(ZG\eta) = \cos\theta \sin\phi \sin\psi - \sin\theta \cos\phi$
$c_3 = \cos(ZG\zeta) = \cos\theta \cos\psi$

Angular Velocities

$p = \dot{\theta} - \dot{\phi} \sin\psi$
$q = \dot{\phi} \sin\theta \cos\psi + \dot{\psi} \cos\theta$
$r = \dot{\phi} \cos\theta \cos\psi - \dot{\psi} \sin\theta$