

# SIMULATION OF VIBRATION IN AUTOMOBILES

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## INTRODUCTION

An automobile is simulated in Advanced Continuous Simulation Language (ACSL) to study the effects of the suspension system and tire characteristics on its vibration and ultimately on passenger comfort. Inputs to the model are the road surface roughness and irregularities; output is the amplitude of vibration sensed by the passenger.

An automobile represents a complex structural system with many degrees of freedom. Two basic models may be employed to study the characteristics of this system; namely vehicle dynamics models and vehicle ride models (1,2). Vehicle dynamics models deal with the dynamic characteristics such as acceleration and translational and rotational motion. A nonrolling, fixed-axis model is based on the rigid body assumption where the automobile as a single rigid mass capable of only sidewise translation and yawing rotation. The rolling of the body (sprung mass) relative to the axles and wheels (unsprung mass) is completely ignored. A more accurate model is a two-mass model with roll and body axes where the sprung mass is free to roll with respect to the unsprung mass.

The second type of model, vehicle ride, deals with the ride quality of automobiles. The ride quality is related directly to the amplitude of vibration sensed and felt by the passenger in the vehicle environment. The vibration of vehicles may be caused by a variety of sources such as road surface roughness and irregularities, aerodynamic forces; and vibration of the engine and transmission. Road irregularity is usually considered the major source of vehicle vibration.

Extensive studies have been done to provide guiding principles for the passenger ride comfort criteria and to define the ride comfort limits (3). In general, these studies relate the vibrational parameters such as displacement, velocity, acceleration, and jerk over the frequency range of interest to the comfort and discomfort limits of the passenger ride. Vehicle ride models have also been developed to obtain the characteristics of automobiles by either determining the principal modes (natural frequencies and mode shapes) of vibration or calculating a time history response for a known excitation of the vehicle. However, these models are simplified to meet the available analytical methods. A two-degree-of-freedom damped or undamped model is typical.

This paper presents a comprehensive vehicle ride model that closely represents an automobile system. The vehicle is modelled as a lumped system in eight degrees of freedom. Eight coupled second-order differential equations with their appropriate geometric constraints are developed based on Newton's second law of motion. The model includes the pitch,

roll, and bounce of the front and rear wheels. It also includes the bounce characteristics of the vehicle seat. However, yawing motion is negligible and is not considered.

This vehicle model is composed of four submodels. The first is the tire and rear axle model (unsprung mass), for which mass, stiffness, and damping characteristics of the tire form the major parameters. The second is the suspension and shock absorber model represented by spring and damper, respectively. The third is the main vehicle body modelled as a rigid body (sprung mass) that can pitch, roll, and/or bounce and the fourth is the seat, in which the seat bounce is characterized by a spring and a damper.

Although the actual vehicle vibration is usually of a random nature due to the road random irregularities, sinusoidal road excitations will be employed in this study. Most of the data used in establishing the available ride comfort criteria have been obtained using sinusoidal input.

In order to obtain a quantitative insight into the effects of the various submodels on vehicle vibration, a parametric design study and analysis are performed. Parameters such as tire stiffness and damping, suspension system stiffness and shock absorber damping are included.

## MATHEMATICAL MODEL

The model under consideration includes the pitch, roll, and bounce of the vehicle as shown in figure 1. The main car body is supported by the suspension, shock absorber, and tire submodels as shown in the figure. The seat characteristics are modelled by a spring and a damper. Based on Newton's second law of motion (4), the following differential equations can be developed;

$$m_1 \ddot{x}_1 = K_1 (y_1 - x_1) + C_1 (\dot{y}_1 - \dot{x}_1) + K_5 (x_5 - x_1) + C_5 (\dot{x}_5 - \dot{x}_1) \quad (1)$$

$$m_2 \ddot{x}_2 = K_2 (y_2 - x_2) + C_2 (\dot{y}_2 - \dot{x}_2) + K_6 (x_6 - x_2) + C_6 (\dot{x}_6 - \dot{x}_2) \quad (2)$$

$$m_3 \ddot{x}_3 = K_3 (y_3 - x_3) + C_3 (\dot{y}_3 - \dot{x}_3) + K_7 (x_7 - x_3) + C_7 (\dot{x}_7 - \dot{x}_3) \quad (3)$$

$$m_4 \ddot{x}_4 = K_4 (y_4 - x_4) + C_4 (\dot{y}_4 - \dot{x}_4) + K_8 (x_8 - x_4) + C_8 (\dot{x}_8 - \dot{x}_4) \quad (4)$$

$$\begin{aligned} \ddot{M}x_{cg} = & K_5 (x_1 - x_5) + K_6 (x_2 - x_6) + K_7 (x_3 - x_7) + \\ & K_8 (x_4 - x_8) + C_5 (\dot{x}_1 - \dot{x}_5) + C_6 (\dot{x}_2 - \dot{x}_6) + \\ & C_7 (\dot{x}_3 - \dot{x}_7) + C_8 (\dot{x}_4 - \dot{x}_8) + K_5 (x_p - x_5) + \\ & C_5 (\dot{x}_p - \dot{x}_5) \end{aligned} \quad (5)$$

$$\begin{aligned} \ddot{J}\dot{\theta} = & -K_5 (x_1 - x_5) (L - L_1) + K_6 (x_2 - x_6) L_1 \\ & - K_7 (x_3 - x_7) (L - L_1) + K_8 (x_4 - x_8) L_1 \\ & - C_5 (\dot{x}_1 - \dot{x}_5) (L - L_1) + C_6 (\dot{x}_2 - \dot{x}_6) L_1 \\ & - C_7 (\dot{x}_3 - \dot{x}_7) (L - L_1) + C_8 (\dot{x}_4 - \dot{x}_8) L_1 \\ & - K_5 (x_p - x_5) L_2 - C_5 (\dot{x}_p - \dot{x}_5) L_2 \end{aligned} \quad (6)$$

$$\begin{aligned} \ddot{I}\dot{\omega} = & -K_5 (x_1 - x_5) W_1 - K_6 (x_2 - x_6) W_1 \\ & + K_7 (x_3 - x_7) (W - W_1) + K_8 (x_4 - x_8) (W - W_1) \\ & - C_5 (\dot{x}_1 - \dot{x}_5) W_1 - C_6 (\dot{x}_2 - \dot{x}_6) W_1 \\ & + C_7 (\dot{x}_3 - \dot{x}_7) (W - W_1) + C_8 (\dot{x}_4 - \dot{x}_8) (W - W_1) \\ & + K_5 (x_p - x_5) W_2 + C_5 (\dot{x}_p - \dot{x}_5) W_2 \end{aligned} \quad (7)$$

$$m_p \ddot{x}_p = K_5 (x_5 - x_p) + C_5 (\dot{x}_5 - \dot{x}_p) \quad (8)$$

where  $L_1$  and  $W_1$  are measured from the c.g. to the right and front sides of the automobile model, while  $L_2$  and  $W_2$  locate the seat model relative to the c.g. shown in figure 1.

Employing the geometric constraints of the model, the displacements  $x_5 - x_8$  of the suspension and shock absorber submodel at its connecting locations with the main car body are given by:

$$x_5 = x_{cg} - (L - L_1) \theta - W_1 \theta \quad (9)$$

$$x_6 = x_{cg} + L_1 \theta - W_1 \theta \quad (10)$$

$$x_7 = x_{cg} - (L - L_1) \theta + (W - W_1) \theta \quad (11)$$

$$x_8 = x_{cg} + L_1 \theta + (W - W_1) \theta \quad (12)$$

and the total energy loss due to the different dampers in the model is:

$$\begin{aligned} E = & \frac{1}{2} C_1 (\dot{y}_1 - \dot{x}_1)^2 + \frac{1}{2} C_2 (\dot{y}_2 - \dot{x}_2)^2 + \frac{1}{2} C_3 (\dot{y}_3 - \dot{x}_3)^2 \\ & + \frac{1}{2} C_4 (\dot{y}_4 - \dot{x}_4)^2 + \frac{1}{2} C_5 (\dot{x}_5 - \dot{x}_1)^2 + \frac{1}{2} C_6 \\ & (\dot{x}_6 - \dot{x}_1)^2 + \frac{1}{2} C_7 (\dot{x}_7 - \dot{x}_1)^2 + \frac{1}{2} C_8 (\dot{x}_8 - \dot{x}_4)^2 \end{aligned} \quad (13)$$

#### MODEL SIMULATION ON ACSL

The purpose of the model is to determine the passenger kinematic quantities; namely its displacement  $x_p$ , velocity  $\dot{x}_p$ , and acceleration  $\ddot{x}_p$  due to a sinusoidal input of the form:

$$y(t) = Y \sin(\omega t) \quad (14)$$

The model simulation is performed on ACSL. The simulation is divided into four major sections (5):

- 1) INITIAL: where the initial conditions and other simulation parameters are set at the beginning.
- 2) DERIVATIVE: where equations 1-14 are modelled with variables updated every integration time step
- 3) DYNAMIC: where logging-in data may occur; and
- 4) TERMINAL: for final calculations.

#### SIMULATION OUTPUT

The simulation can be used to evaluate the effects of various system parameters on the passenger response  $x_p$ . Different parametric studies have been performed to study the effects of the tire stiffness  $K_1 - K_4$ , the tire damping  $C_1 - C_4$ , the suspension stiffness  $K_5 - K_8$ , and the damping of the shock absorbers  $C_5 - C_8$ . Figures 2-7 illustrate these effects. In all the figures, the following data has been used:

$$K_1 = K_2 = K_3 = K_4 = 1000 \text{ lb/in}$$

$$K_5 = K_6 = K_7 = K_8 = 100 \text{ lb/in}$$

$$C_1 = C_2 = C_3 = C_4 = 50 \text{ lb.sec./in}$$

$$C_5 = C_6 = C_7 = C_8 = 200 \text{ lb. sec/in}$$

$$K_5 = 500 \text{ lb/in}, C_5 = 50 \text{ lb.sec/in}$$

$$y_1 = y_2 = y_3 = y_4 = 1 \sin 2 \pi t$$

$$m_1 = m_2 = m_3 = m_4 = 100 \text{ lb}$$

$$M = 3500 \text{ lb}, m_p = 200 \text{ lb}$$

$$L = 110 \text{ in}, L_1 = 50 \text{ in}, L_2 = 0$$

$$W = 100 \text{ in}, W_1 = 50 \text{ in}, W_2 = 0$$

$$r_{\text{roll}} = 48 \text{ in}, r_{\text{pitch}} = 72 \text{ in}$$

where  $r$  is the radius of gyration. The initial conditions were taken to be zeros.

Figure 2 shows the effect of the tire stiffnesses on the passenger amplitude. The displacement  $x_p$  decreases by increasing the stiffnesses of the tires. The total energy loss in the model is shown in figure 3 for two different values of the stiffnesses of the tires. That energy loss includes losses due to both the shock absorber damping and the tire viscous damping characteristics. As shown, the stiffer the model, the less energy loss occurs due to the decrease in the model vibration. In figure 4, the passenger amplitude increases with the increase of the suspension stiffnesses. Figure 5 shows that high duty shock absorbers make the passenger feel more of the automobile vibration.

ACKNOWLEDGEMENT

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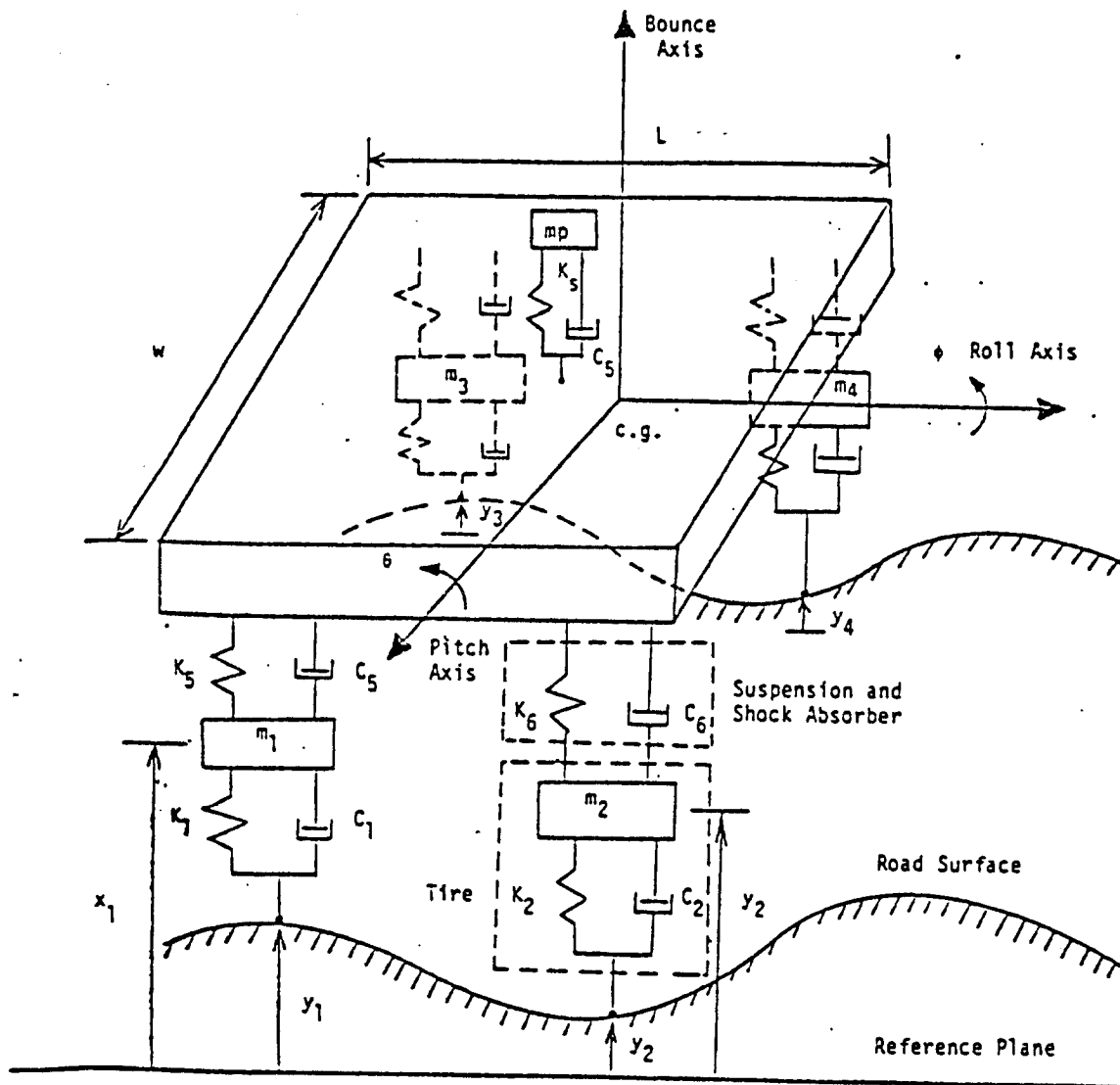


Figure 1 Automobile Model

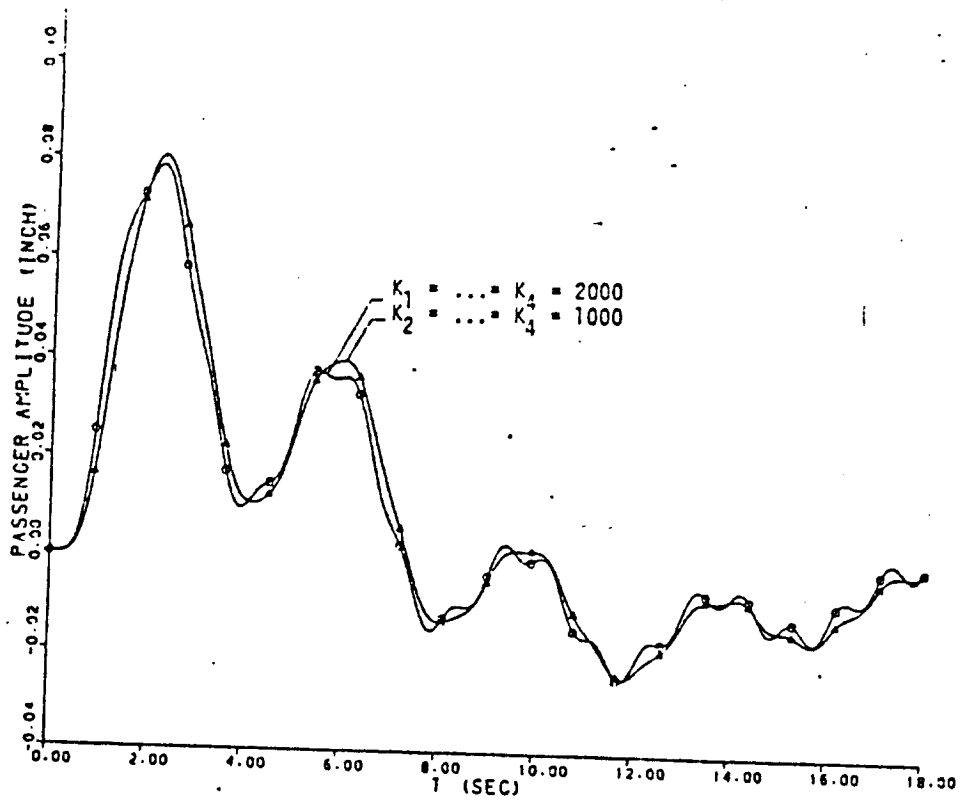


Figure 2 Effect of Tire Stiffness on Passenger Amplitude

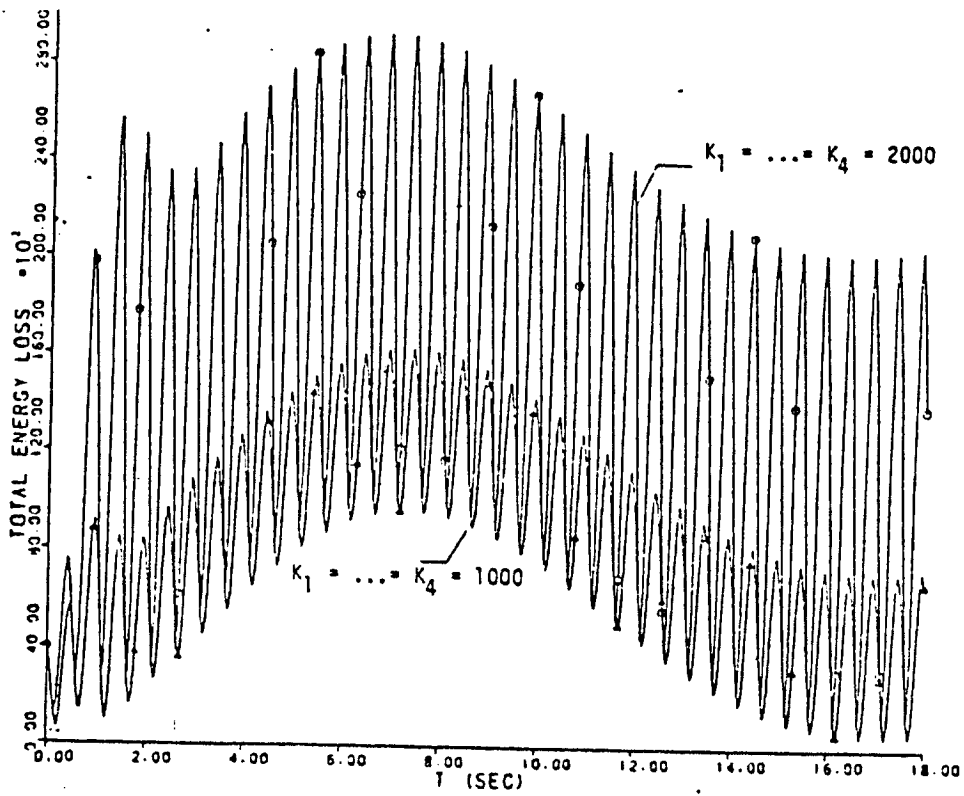


Figure 3 Effect of Tire Stiffness on Energy Loss

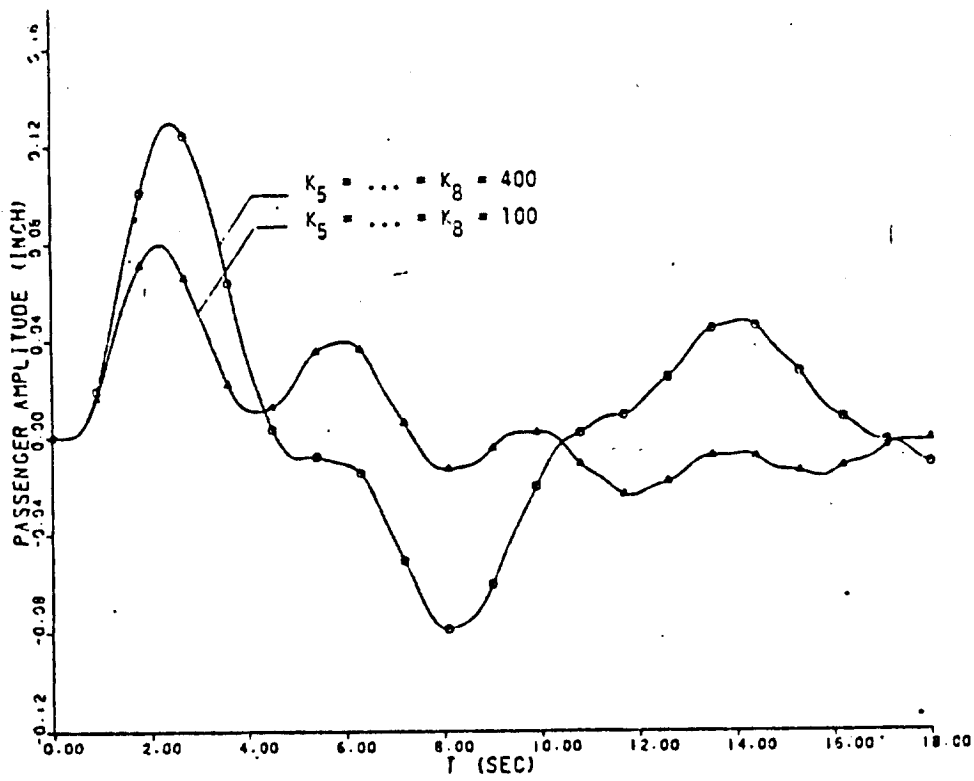


Figure 4 Effect of Suspension Stiffness on Passenger Amplitude

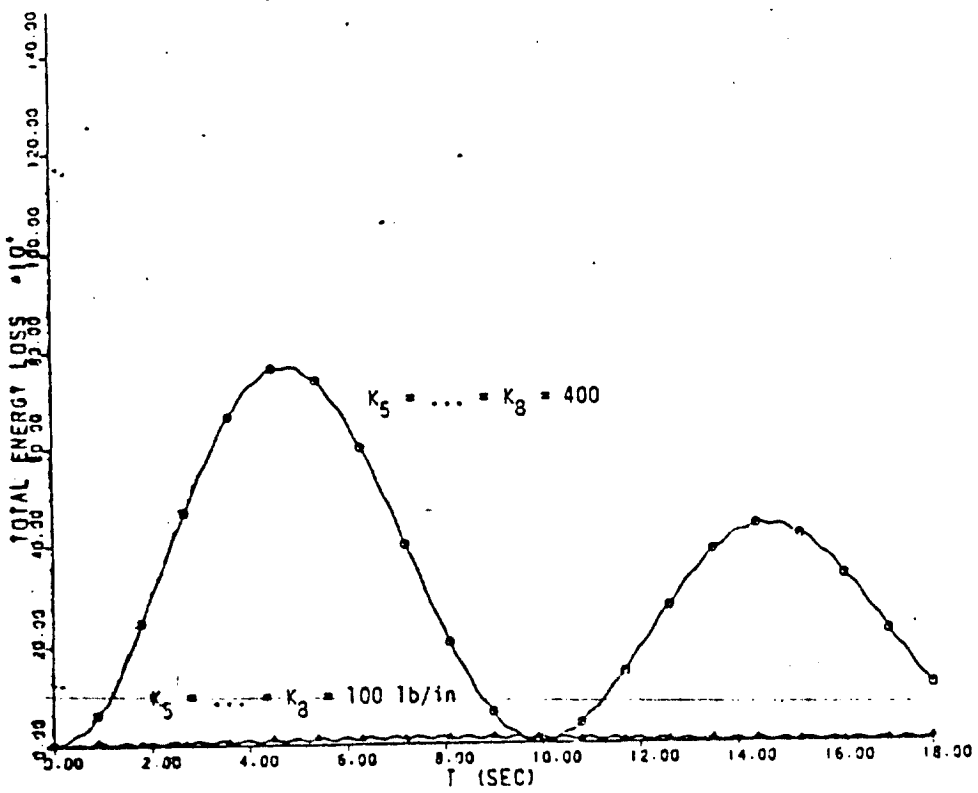


Figure 5 Effect of Suspension Stiffness on Energy Loss

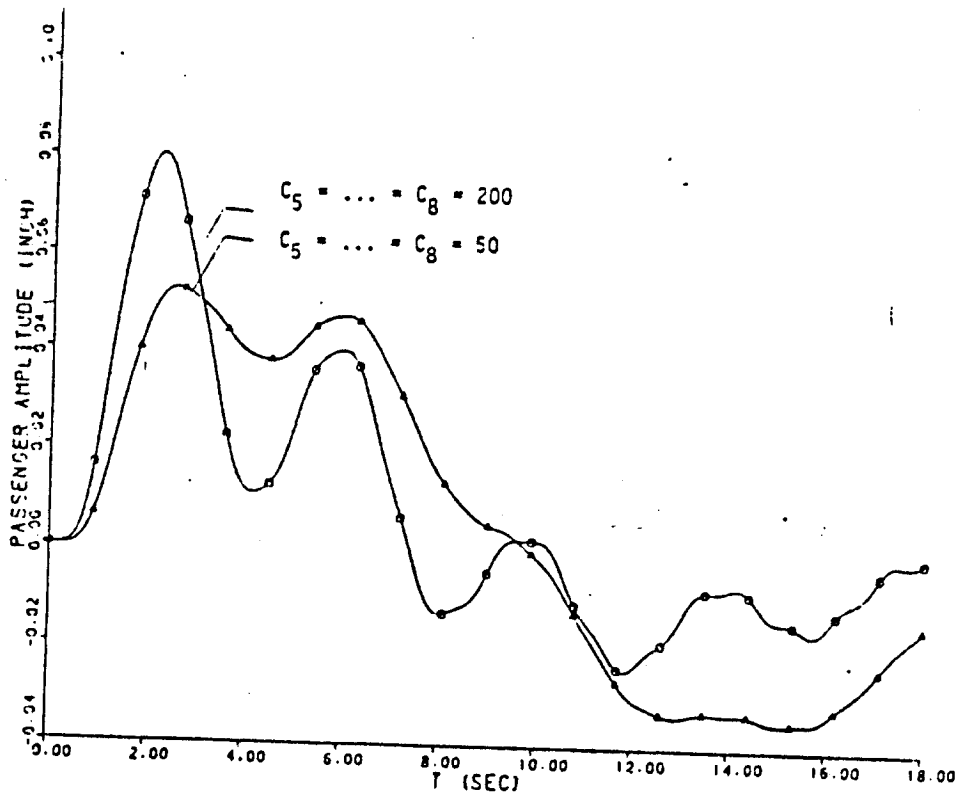


Figure 6 Effect of Damping of Shock Absorber on Passenger Amplitude

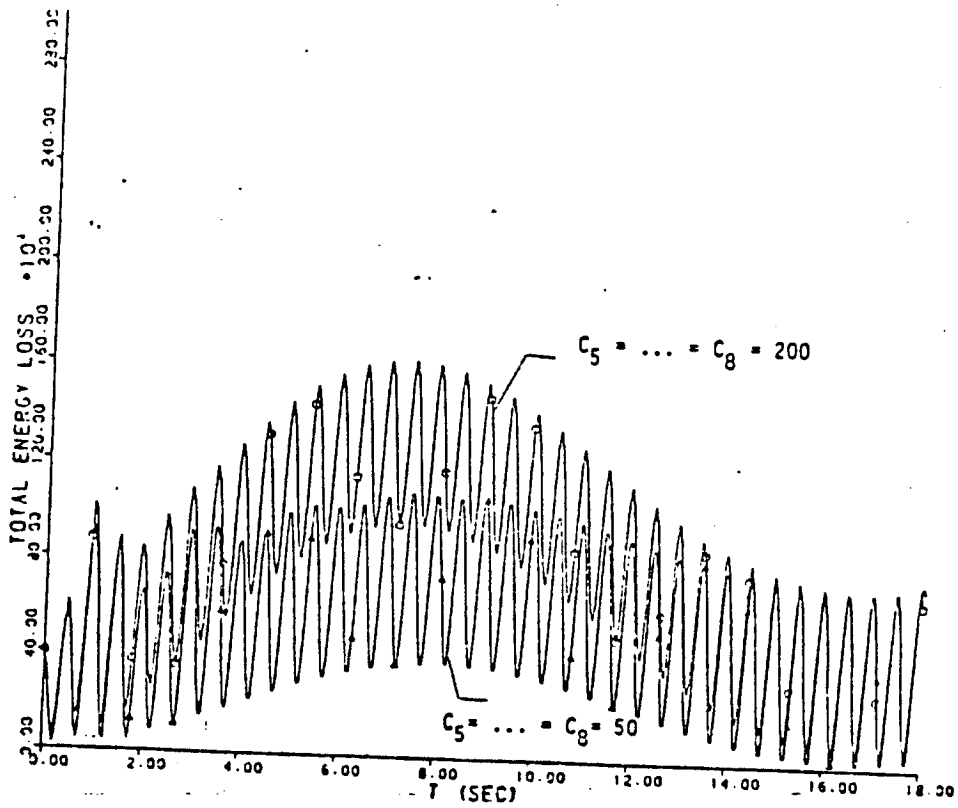


Figure 7 Effect of Damping of Shock Absorber on Energy Loss